Teaching with Technology in Undergraduate Mathematics

Workbook

Developed through a partnership across three Ontario universities, this set of interactive modules offers self-paced and differentiated professional development to help Instructors and Graduate Students build capacity for enacting innovative, inclusive, and digitally enhanced pedagogy for undergraduate mathematics.

This project is made possible with funding by the Government of Ontario and through eCampusOntario's support of the Virtual Learning Strategy. To learn more about the Virtual Learning Strategy visit: https://vls.ecampusontario.ca

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Course Overview

As part of the Government of Ontario's Virtual Learning Strategy (VLS) and facilitated by eCampus Ontario, we are excited to present this interactive and multi-modal e-course **Teaching with Technology in Undergraduate Mathematics.** Designed to help Instructors, Tutors, & Teaching Assistants enhance and apply skills for leveraging innovative and interactive digital technologies to support inclusive learning and student achievement.

Digital technologies offer educators new possibilities for how to engage, interact with, and support students.

This course explores research-based strategies for how accessible and available digital technologies can be leveraged to enhance students' mathematical experiences, learning, and understanding.

Through your participation in this course, you will:

1. Develop fluency in the purposeful uses of different digital technologies for representing and communicating mathematics in undergraduate education.
2. Enhance pedagogical approaches for fostering rich mathematical experiences through the uses of interactive digital technologies and mathematical questioning.
3. Build capacity for enacting inclusive pedagogies that incorporate digital technologies into online, blended or in-person undergraduate mathematics learning environments.
4. Consolidate ideas from Modules 1 - 4 and apply them to the design and development of interactive, digitally-enhanced mathematics learning environments.

Self-paced Learning

We designed this course with flexibility in mind; learn in your own time and by your own schedule. Course material can be reviewed, rewound, revisited as many times as required. Where relevant, we also share supplementary resources, research literature, and online community hubs so that learners can explore and supplement their course experiences with a variety of connections.

Differentiated Experience

To complete this course, you will respond to the workbook questions included in this booklet. Questions are differentiated based on your context, and you will have the option to choose amongst questions aimed at Instructors or Teaching Assistants. Questions focus on applying course ideas and technologies to your personal teaching context, reflecting on those experiences, and developing new transferable competencies for undergraduate mathematics teaching.

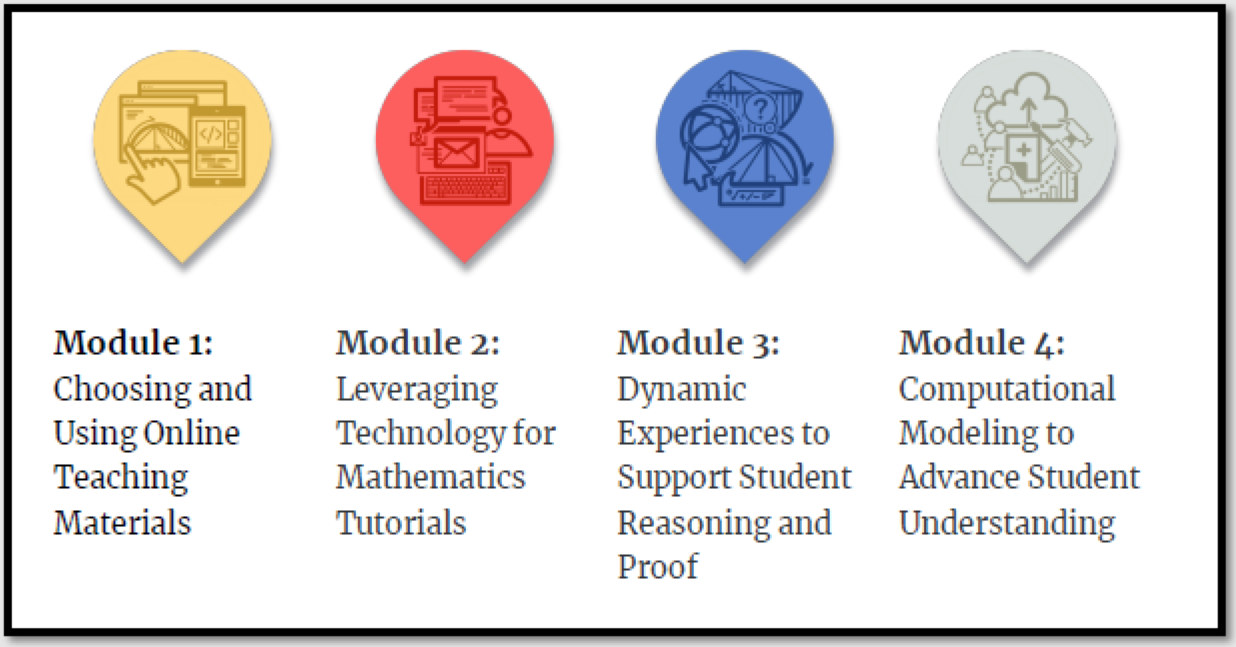
Multiple Course Pathways

Whether you are an Instructor, Teaching Assistant, or Graduate Student in Math Education, there is something special in this course for you. The course offers multiple pathways through targeted activities, resources, and more, and you will be able to choose the pathway that best fits your learning goals and needs.

Instructors will engage in activities and workbook problems that invite them to reimagine how to design and deliver their courses, while developing the digital fluency and pedagogical expertise that every 21st century educator needs. TAs and Graduate Students will engage in a subset of these activities, with workbook questions that will help them enhance their practice and optimize their time.

The accompanying Resource Book is designed to supplement the e-course, as well as be used as a stand-alone resource. It includes two sections: Strategies for Responding to Student Needs, and Strategies for Fostering Mathematical Thinking. A master list of supplementary resources and references is also included.

This course consists of four modules:



The **workbook** and **resource book** serve as companion guides for this e-course. The workbook is designed to help foster connections across modules for a holistic sense of how technologies can be purposefully integrated to enhance your practice. Through your engagement with the course you will develop technical skills, pedagogical awareness, and transferable competencies that will equip you for teaching effectively with technology in in-person, blended or online learning environments. You can record your written responses directly in this workbook. <u>Click on the pins in the above image to jump to the workbook questions for each activity</u>.

Module 1: Choosing & Using Online Teaching Materials

Activity 1
Teaching with Video Podcasts: An Example in Linear Algebra

Activity 2
Dynamic Visualization in Calculus

Activity 3
Computational Modeling as a Teaching Aid

This module explores the uses, limitations, and benefits of incorporating different digital technologies into lecture based teaching, and it highlights what instructors need to consider in order to ensure that the technologies they use are accessible and useful for students.

Differentiated Pathways

For TAs and Graduate Students:

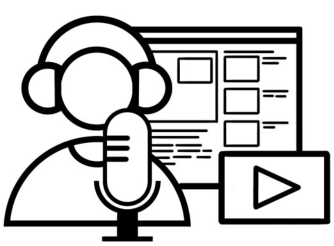
- **Dynamic Geometry Pathway:**
  - Complete workbook questions for Activities 1 and 2.
- **Computational Modeling Pathway:**
  - Complete workbook questions for Activities 1 and 3.

For Instructors:

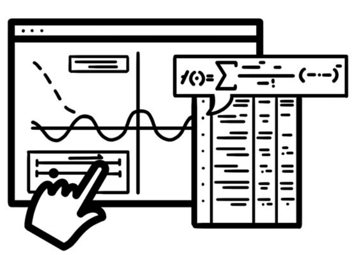
- Complete workbook questions for Activities 1-3
- Choose an area of focus and apply ideas and resources with the intent to augment one of your current lectures (see the final question in the workbook).

Go to:

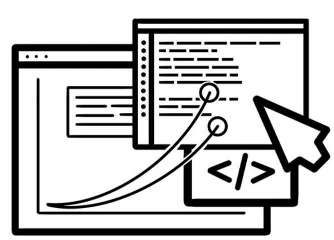
Activity 1



Activity 2



Activity 3



Workbook Questions for
Activity 1: Integrating Video Podcasts into Lessons

1. Find and explore Grant Sanderson's website and briefly summarize his licensing policy. Does it align well with your particular use case and learning management system?

2. When we want to share content online, we need to be sure that we share it in ways that will be accessible for our students. Find a video from the series The Essence of Linear Algebra and check its captions. Do you think they were automatically generated or hand written? Are they moving too quickly? Can you follow the content of the video by reading the captions alone?

3. When sharing a video online we are tempted to watch the video in real time with students. What issues might arise? How would you resolve those issues?

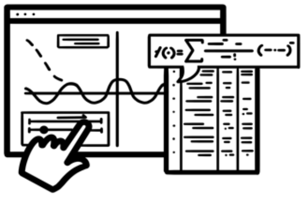
4. Suppose that you're teaching an introductory linear algebra course. Consistently, you've used the notation $\overline{e}_1$ and $\overline{e}_2$ for basis vectors in the plane. On your course management website, you've included a link to 3Blue1Brown's Essence of Linear Algebra Course. A student from the course comes to office hours having watched the video Linear combinations, span, and basis vectors and asks whether $\hat{\imath}$ and $\hat{\jmath}$ will be on the upcoming midterm.

   a. How would you respond to this student?

   b. In your lectures, you have not used this notation. What kinds of questions do you anticipate your students will have after watching this video?

c. What are some different strategies you could use to curate video podcasts, either with student participation or without? What are some of the benefits or drawbacks of each approach?

5. When selecting videos, instructors need to be mindful of constraints on students' time and energy. Describe what value could be gained from sharing a series like the Essence of Linear Algebra with your students. How would that gain compare to other modes of delivery such as lecturing or asking students to do readings?



Workbook Questions for Activity 2: Dynamic Modeling and Visualization in Calculus

1. In which part of the lesson should the GSP and GeoGebra applets be brought to students' attention and why would it be most beneficial to introduce the applets at this stage of students' learning? Please explain.

2. In what ways do the applets differ with respect to how they portray important ideas about Taylor Series?

3. What implications do these differences have on how each applet could be used to facilitate learning?

   a. What are their educational benefits of each applet?

   b. What are the educational limitations of each applet?

4. What questions can be asked to harness students' understanding of sin(x) or cos(x) as a series, and extend this perspective to a broader world of functions? (Hint: for instance, a question might suggest that they relate properties of graphs (such as even, odd) with the degrees of Taylor polynomial approximations; or, ask: When is it ok to replace cos(x) by x?

5. Consider a situation where, at some arbitrary point, a function is poorly approximated by a Taylor polynomial. Which applet would you choose to demonstrate that the approximation is indeed poor? What changes would you make to improve it?

Workbook Questions for
Activity 3: Computational Modeling as a Teaching Aid

1. Why are the two curves, Euler and algebraic, different?
   a. What responses do you anticipate students might give to this question?

   b. Find the lines in the code which make the Euler graph terminates at $t = 5$.

   c. How many times does the code go through the loop?

2. Clearly, Euler's curve moves farther and farther from the algebraic curve as t moves away from the initial condition. How can we change the code to make the two curves come closer to each other?

3. In this case, we were lucky, as we could compute the exact solution (called algebraic). How would you verify that your code gives a good approximation of the solution if you did not have the exact solution?

4. How would you modify the code to study the equation $y' = te\hat{}(-1/y\hat{}2)$, y (0) =1? Note: This equation cannot be solved in a closed form (i.e., as a compact formula; it can be solved as a power series)

   a. Ask students to suggest strategies for testing the approximate solution they obtained (remind them that Python will produce both numeric solution and also its graph). What do you think they will suggest, related to the checking geometric properties of the solution?

b. If you ask that they write additional codes to test their solution, what do you think they will suggest?

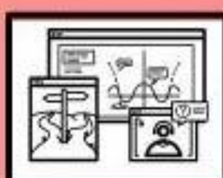
For Instructors:

Reflect on the technology you explored in this module. Select one - video podcasts, dynamic geometry software, or computational modeling software - and explore some of the available resources that are applicable to your teaching. Exemplify (either in a written, audio, video, or digital artefact) how you might incorporate the resource(s) into your lectures. Explain what you will need to do in order to make sure the resource is accessible and useful for your students.

Module 2: Leveraging Technology for Mathematics Tutorials

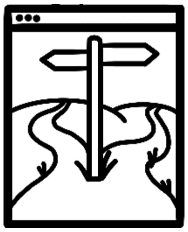
Activity 1
Leading Effective Discussions

Activity 2
Leading Effective Discussions Online



Activity 3
From Lecture to Active Learning

This module includes interactive video simulations that model and respond to common issues which arise in tutorial settings and other student-centered learning environments. It examines questioning strategies and communicational technologies that can be leveraged to foster student engagement and understanding.



Differentiated Pathways

For TAs and Graduate Students:

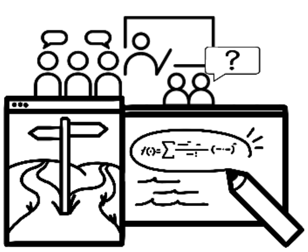
- **Dynamic Geometry Pathway:**
  - Complete workbook questions for Activities 1 and 2.
- **Computational Modeling Pathway:**
  - Complete workbook questions for Activities 1 and 2.

For Instructors:

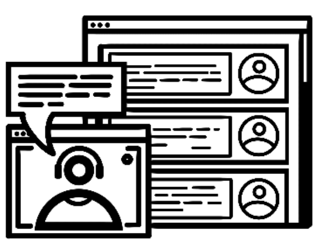
- Complete workbook questions for Activities 1-3
- Apply a critical eye toward ways mathematical discussion and debate can be used to engage students and foster transferable competencies in deductive reasoning. Explore implications for tutorials and strategies for supporting TAs.

Go to:

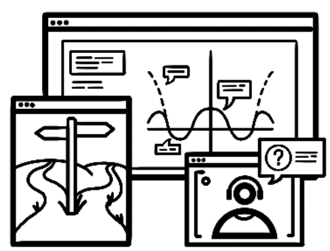
Activity 1

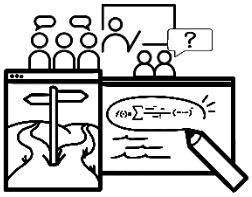


Activity 2



Activity 3





Workbook Questions for
Activity 1: Leading Effective Discussions

To complete the questions for this activity, you will need to engage with the interactive video simulation found on the course website, as well as consult the Resource Book included with this educational guide.

1. The video simulation ended with our TA struggling to make sense of a student's question. Imagine yourself in a similar situation - you have no idea what your student is trying to ask, you are concerned it will cause other students to become confused, and you do not want to embarrass or dismiss the student who is clearly in need of assistance. In such a situation, it is useful to have a set of questions in your back pocket that can help you unpack and understand the question and steer the class toward the right track. Create a set of questions that you could pose to the student and class that would help navigate and resolve this situation. Please explain your intentions behind each question.

2. The following task is adapted from Zazkis & Marumur (2018).

   a. Let $f, g: \mathbb{R} \to \mathbb{R}$ be functions such that $f\big(g(x)\big) = x$ for every $x \in \mathbb{R}$.
      (i) Prove that $g(x)$ is injective (one-to-one).
      (ii) Prove that $f(x)$ is surjective (onto).

   In the space below, record what techniques you used to prove each statement.

   b. What questions or challenges do you think might arise in class if you asked your students to do the same task?

c. Using the same functions from part a, prove or disprove: $g(x)$ is invertible. What technique did you use?

d. Consider the following example:

$$g(x) = 2^x, f(x) = \begin{cases} \log_2 x, x > 0 \\ 1, x \leq 0 \end{cases}.$$

A student working with this example made the following statement: "I remember learning that exponential functions such as $g(x) = 2^x$ are invertible, and that the inverse is $\log_2 x$." How would you respond?

3. Consider a theorem from your teaching which can be disproved with a counter example.

   a. Create a set of examples that appear to satisfy the theorem but which really do not. Come up with as many examples as you can.

   b. What is similar and what is different about the examples in your set?

c. Come up with one more example. Try to make it notably different from your other examples. (What makes it different?)

d. You now have an example space of “covert” counter examples. Reflect on how exploring covert counter examples can advance student understanding of the theorem you chose.

Workbook Questions for
Activity 2: Leading Effective Online Discussions

To complete the questions for this activity, you will need to engage with the interactive video simulation found on the course website as well as consult the Resource Book included with this educational guide.

1. The video simulation ended with our TA realizing that there was someone who was not able to make sense of the lesson.

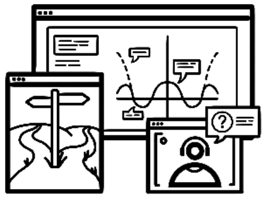
   a. Create a brief video explanation of this proof using software such as Explain Everything. Try to follow some of the best practices for creating video podcasts (i.e., keep it short and clear) and don't worry about polish. Students appreciate authenticity; being genuine and kind is more important and impactful than creating a "perfect" video.

   b. Videos are also a useful way of giving quick feedback on student work. Use annotating software such as VideoAnt, H5P, or other, to create video feedback for students in your tutorial. Try this out the next time students ask you to explain what they did wrong on a test or assignment.

   c. Reflect on these two uses of video and discuss the pros and cons of each.

2. Using the technology of your choice, create a poll to solicit questions in advance for one of your future tutorials. What questions will you include on the poll (and why) and how will you distribute it to students?

3. On the e-course website, you will find a selection of challenging questions that have come up in first year classes and tutorials; they are presented as an interaction during one-on-one office hours between you and a student. Choose one of the scenarios and follow the prompts on the e-course website to resolve the student's confusion.



Workbook Questions for
Activity 3: From Lecture to Active Learning

1. Consider how you might use pop culture media, such as popular podcasts or videos, to launch a discussion-based mathematical exploration.

    a. Consider the benefits and challenges of bringing podcasts such as RadioLab into a mathematics class, from both a student's and an instructor's perspective. Respond to this question in general, and with particular reference to the example discussed in this activity.

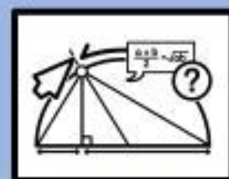
b. Share an example of the media you are considering and outline how students might engage with it. Would this be an exploration done in class or in tutorial? If in class, how could you use tutorials to supplement the exploration? If in tutorial, how would you support your TA in handling the discussion?

c. What mathematical discrepancies or inaccuracies or misleading content did you notice in the pop culture media you selected? How could you leverage this to foster a critically-minded mathematical discussion?

2. Consider the relevance of critically-minded debate for the advancement of mathematical reasoning and proof. How might the inclusion of ambiguous or contentious real-world issues in mathematics courses be leveraged to help contest the common misconception that proof is a formal, pedantic or unimaginative exercise of rules and minutiae?

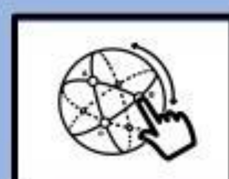
3. Shifting from lecture-based to active learning environments can open up a lot of new possibilities and challenges, including with respect to how tutorials can be used. This is particularly true in an online setting. Consider what might need to change in how you organize tutorials and support TA's in a fully online course in which lectures are not the main form of instruction. Please explain.

Module 3: Dynamic Experiences to Support Student Reasoning & Proof



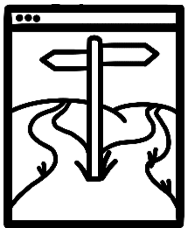
Activity 1
Grasping Proof with Dynamic Visualization

Activity 2
Responding to Student Emails with DGS



Activity 3
Theory in Motion: A New Look at Girard

This module includes activities aimed at supporting student-centered teaching strategies, as well as an interactive video simulation that highlights ways DGS can be responsively to respond to student questions and foster student thinking. It considers how students' learning experiences can be redefined through experimenting with DGS and it examines what modifications in your teaching approaches might be needed in order to support this new learning.



Differentiated Pathways

For TAs and Graduate Students:

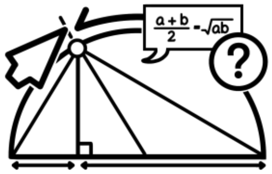
- **Dynamic Geometry Pathway:**
  - Complete workbook questions for Activities 1 and 2.
- **Computational Modeling Pathway:**
  - See Module 4.

For Instructors:

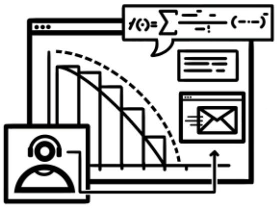
- Complete workbook questions for Activities 1-3
- Apply a critical eye to compare and contrast how student learning and mathematical interactions are impacted by uses of Dynamic Geometry Software or Computational Modeling.

Go to:

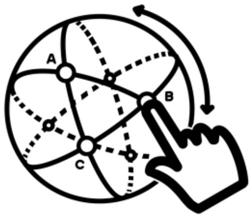
Activity 1

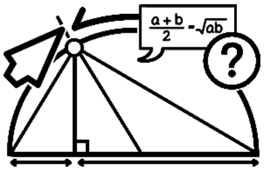


Activity 2



Activity 3





Workbook Questions for
Activity 1: Grasping Proof with Dynamic Visualization

1. Revisit the three applets that depicted the AM-GM inequality. Compare and contrast the three options in terms of how they capture and portray key ideas. Consider both the explanatory power of the applets and their mathematical precision and generalizability. Discuss the benefits and limitations of each applet.

2. Put yourself in a student's shoes and consider how each applet illustrates the AM-GM inequality by attending to the questions posed below. How do these questions support students' experiences with the applets? Anticipate where students will struggle and suggest further questions that could help them better understand.

GeoGebra applet:

Given that the shaded right-angle triangles have side lengths a and b:

i. What is the area of the large square?
ii. Is C'D'E'F' a square? How do you know?
iii. As point P moves, what happens to the right-angle triangles?
iv. As point P moves, what happens to the area of the center square?
v. What is the relationship between the area of the large square and the area sum of the triangles?
vi. How is this relationship related to the AM-GM inequality?

Desmos applet:

Based on what is given in the diagram

i. What can you say about the blue and green triangles?
ii. What can you say about the areas of the two triangles?
iii. As point D moves, what happens to the area of rectangle ABCD?
iv. As point D moves, what happens to the area sum of the two triangles?
v. How is AM represented in this context?
vi. How is GM represented?
vii. How are the AM and GM related?

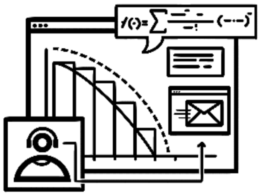
GSP applet:

i. In the semicircle, OQ is the radius. What is the length of OQ in terms of a and b?
ii. In the semicircle, if AC=a, CB=b, how do you know PC=$\sqrt{ab}$?
iii. As point P moves, what is the relationship of PC to radius OQ?
iv. How is the AM-GM inequality illustrated through the diagram?

How do these questions support students' experiences with the applets? Anticipate where students will struggle and suggest further questions that could help them better understand.

3. Compare and contrast the three applets in terms of how they each capture and portray key ideas. Consider both the explanatory power of the applets and their mathematical precision and generalizability. Discuss the benefits and limitations of each applet.

4. Identify which applet you think might be the most helpful for a student and what additional explanations you could provide to guide students toward a formal proof. Please explain your thinking.



Workbook Questions for
Activity 2: Responding to Student Emails with DGS

To complete the questions for this activity, you will need to engage with the interactive video simulation found on the course website, as well as consult the Resource Package included in this educational guide.

1. The video simulation ended with our TA inviting students to office hours to go over 3D rotations. In his email, the TA directed students to a video explanation. Search the web for videos that explain 3D rotations and give careful thought to the educational experience of each video. Choose the video which you think would be most appropriate given the TA’s and answer the following questions:

   a. What factors did you consider when making your choice, and why?

b. How will students engage with the video? Will they need to watch the entire video or only selected parts? Please explain.

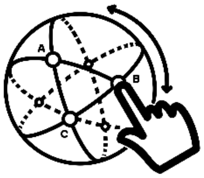
c. Consider the video from a student's perspective. What are some of the "thinking milestones" that students will need to reach in order to make sense of the video? Create a set of 3-5 questions that you could pose to help scaffold students' attempts to reach these milestones.

2. During the interactive activities of the video simulation, you created a walk through of a DGS explanation of the function $f(x) = 2xexp(-0.4x) + 3exp(0.2x)$. Consider how you could use an Interactive Whiteboard and (i) real-time collaboration, and (ii) screencast software, to explain key properties of the function and its graph.

3. Consider a question that you received recently from a student in one your tutorials and craft a response to your student using technologies and techniques from this course. Alternatively, you may choose a question from a selection of student emails that are included on the course website. Using the screencast software of your choice, create and record a walk through of your response. Narrate your recording and explain how you are using the technology and techniques to support and advance student thinking. Submit your walk through as a video link.

For Instructors:

This activity is similar to one you will complete in Module 4, yet there are key differences that reflect the usability of the two technologies - Dynamic Geometry Software and Computational Modeling. Anticipate differences between how the technologies can be used to communicate with students, augment their learning, and purposefully direct attention toward important mathematical ideas. (If you have already completed Module 4, compare and contrast your experiences, and discuss your reflections with examples.)



Workbook Questions for
Activity 3: Theory in Motion: A New Look at Girard

1. The exploration of Girard's Theorem highlighted the complementary roles that different representations can play in fostering student reasoning and understanding. Drawing on your own practice, consider the theorems you are (will be or have been) teaching and make note of which ones could be supplemented with a DGS applet. What features of a theorem make it well-suited for a DGS exploration? Please explain.

2. Of the theorems you identified in question 1, select one that students tend to really struggle with. Review the proof with a close eye toward which parts of the written / verbal proof are difficult to grasp.

   a. Are there diagrams or sketches that could help explicate the tricky bits? Draw them on a blank piece of paper.

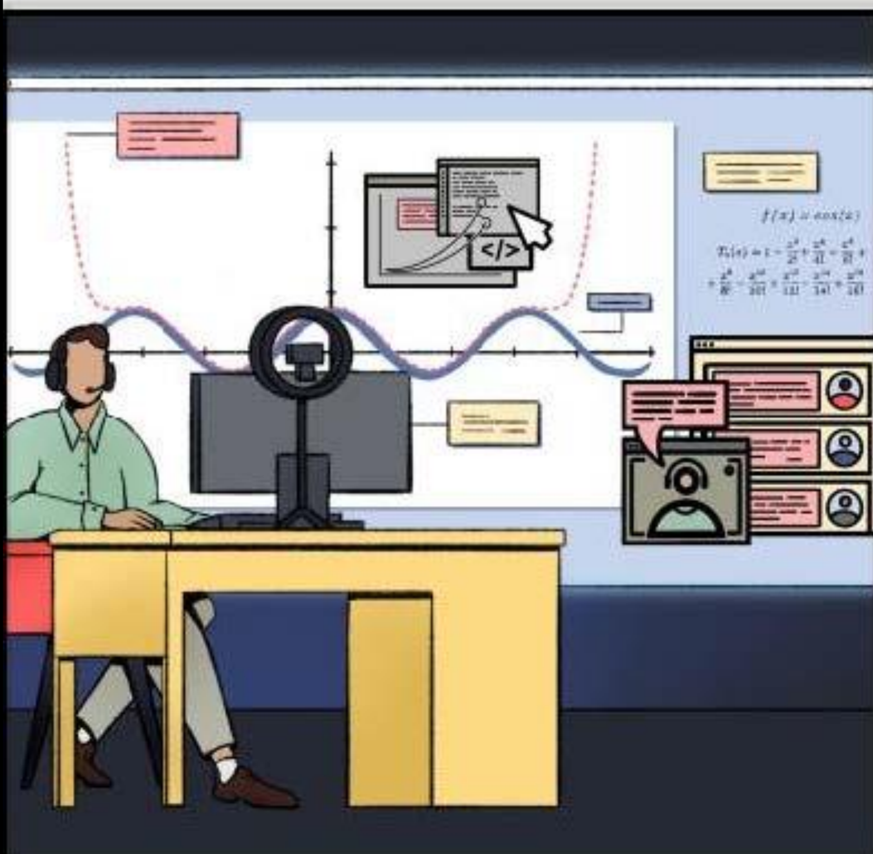
b. Search the GeoGebra repository. Does an applet already exist that depicts this theorem? If so, experiment with the dynamic model and investigate some of the trickier bits of the theorem. (If not, choose a different theorem from your course for which an applet already exists and bookmark your other theorem for later.) Does the applet coincide with the diagrams you drew?

c. What questions could you pose to students to help them investigate this theorem?

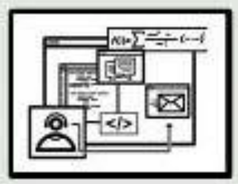
d. How might the experience of interacting with the GeoGebra applet impact students' understanding of specific aspects of this theorem and its proof?

3. Choose another theorem from your teaching. This time select one that does not have a pre-made GeoGebra applet. Using the Dynamic Geometry Software of your choice, create an applet that models the theorem. Using the screencast software of your choice, create a brief tour of your applet and its explanatory and exploratory features.

Module 4: Computational Modeling to Advance Student Thinking



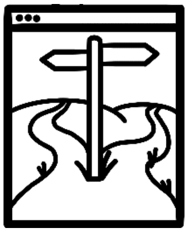
Activity 1
Modeling Linear Algebra with Python



Activity 2
Responding to Student Emails with Python

Activity 3
Pythonize a Lecture: An Example in Finite Sums

This module includes activities aimed at supporting student-centered teaching strategies, as well as an interactive video simulation that highlights ways computational modeling, specifically Python programming, can be responsively to respond to student questions and foster student thinking. It considers how students' learning experiences can be redefined through experimenting with computational modeling and it examines what modifications in your teaching approaches might be needed in order to support this new learning.



Differentiated Pathways

For TAs and Graduate Students:

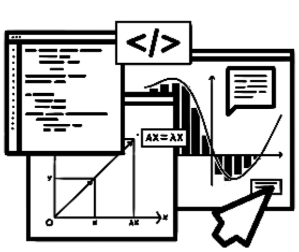
- **Dynamic Geometry Pathway:**
  - See Module 3.
- **Computational Modeling Pathway:**
  - Complete workbook questions for Activities 1 and 2.

For Instructors:

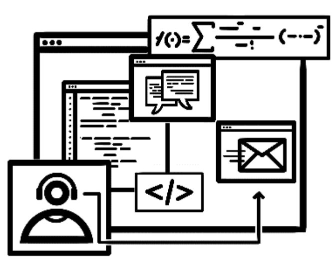
- Complete workbook questions for Activities 1-3
- Apply a critical eye to compare and contrast how student learning and mathematical interactions are impacted by uses of Dynamic Geometry Software or Computational Modeling.

Go to:

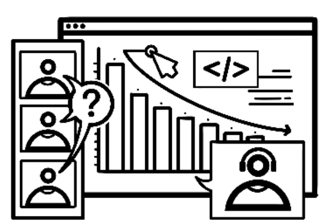
Activity 1

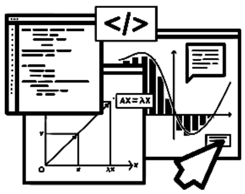


Activity 2



Activity 3





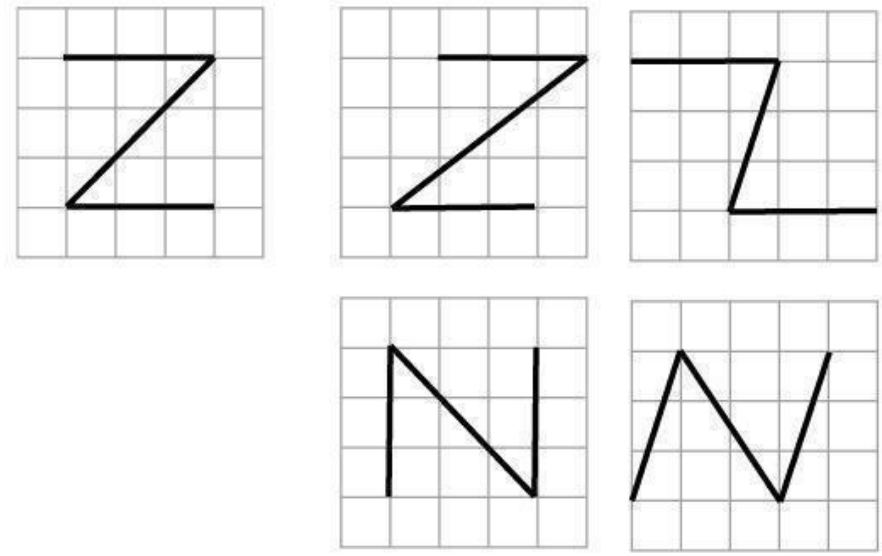
Workbook Questions for Activity 1: Modeling Linear Algebra with Python

1. Think about expanding your code to use in your classroom. What do you anticipate your students will struggle with most when you discuss linear transformations?

   For instance, students are familiar with graphing functions and describing their features (extreme values, concavity, etc.). However, they have lot less experience visualizing linear transformations and their properties. Think about the ways you can use Python code to help your students. Which properties of linear transformations would you choose to discuss, and why?

2. Having Python at your disposal, think about what special matrices, which align with the learning objectives of your course, you can use to illustrate various properties of linear transformations. Keeping in mind the difficulties you anticipate your students will face, how would you (re)design the code, and how would you use it in class?

3. Consider the question suggested by the picture below: a given character (letter Z) is transformed, and we have to find the transformation that produced each output.



a. How could you use this exercise to further enhance your students' understanding of the geometry of transformations?

b. Could you envision a small group activity? How would you connect the discussion after the groups present their answers with the Python code that you have developed/modified from the previous exercises?

c. Where do you anticipate students might have the most difficulty? How will you support them through this difficulty?

The following group of questions relates to eigenvalues and eigenvectors

4. Think about the first example that you would discuss - what matrix would you choose, and why? What would you suggest that your students do before running the code with that matrix?

   Hint: play to figure out what the transformation does, and then think about the geometric meaning of eigenvectors.

5. Next, you wish to design a sequence of drill/routine questions to reinforce thinking about transformations as a way of visualizing eigenvalues and eigenvectors. You will suggest that your students go over all of them. Which 2 by 2 matrices would you select? Remind students that they can use code you gave them to check their work.

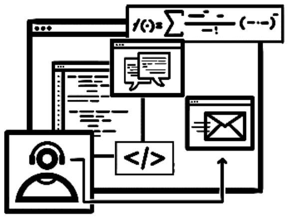
6. Once your students are comfortable with the concepts (say, they know how to do questions (1)-(3)), think of extension to transformations in space. For instance, discuss the matrix and remind them that a bottom right 2 by 2 matrix is a rotation in a plane.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix}$$

a. What further hint(s) would you give to your students (if any) to help them describe the eigenvalues of this matrix?

b. To enhance students' geometric experience working in space (3D), what possible software or apps could you use?

7. Think of other 3 by 3 matrices that you can use to enforce your students' understanding of 3D transformations, in particular their eigenvectors. Would you ask them to modify the code above to check their answers? If your answer is "no," think of replacing coding, in this case, with a ready-made app.



Workbook Questions for
Activity 2: Responding to Student Emails with Python

To complete the questions for this activity, you will need to engage with the interactive video simulation found on the course website as well as consult the Resource Package included in this educational guide.

1. The video simulation ended with our TA inviting students to office hours to go over Simpson's Rule. In his email, the TA directed students to a video explanation. Search the web for videos that explain Simpson's Rule and give careful thought to the educational experience of each video. Choose the video which you think would be most appropriate given the TA's and answer the following questions:

    a. What factors did you consider when making your choice, and why?

b. How will students engage with the video? Will they need to watch the entire video or only selected parts? Please explain.

c. Consider the video from a student's perspective. What are some of the "thinking milestones" that students will need to reach to make sense of the video? Create a set of 3-5 questions that you could pose to help scaffold students' attempts to reach these milestones.

2. During the interactive activities of the video simulation, you tried out two different technologies for computing and explaining the limit of $(x^2 + x + 1)\sin(2\pi x)$ as x approaches infinity. Reflect on and compare your experiences using Python and Interactive Whiteboards to teach and explain ideas.

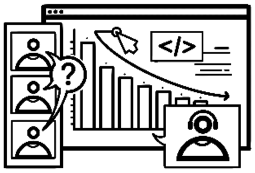
   a. What is similar and what is different about how each approach captures and conveys important ideas related to limits?

   b. Discuss how you might integrate one, or both, of these technologies for communicating with students asynchronously (such as through email).

3. Consider a question that you received recently from a student in one your tutorials and craft a response to your student using technologies and techniques from this course. Alternatively, you may choose a question from a selection of student emails that are included on the course website. Using the screencast software of your choice, create and record a "walk through" of your response. Narrate your recording and explain how you are using the technology and techniques to support and advance student thinking. Submit your walk through as a video link.

For Instructors:

This activity is similar to one you completed in Module 3, yet there are key differences that reflect the usability of the two technologies - Dynamic Geometry Software and Computational Modeling. Compare and contrast how the technologies were used to communicate with students, augment their learning, and purposefully direct attention toward important mathematical ideas. Discuss your reflections with illustrative examples. (If you have not completed Module 3 yet, anticipate the similarities and differences between how the technologies can be used for these goals.)



Workbook Questions for
Activity 3: Pythonize a Lecture: An Example in Finite Sums

1. Consider the content covered in this lecture series. In what parts of the lectures could Python be integrated? Anticipate what requisite coding knowledge would be needed for these purposes

2. Consider Section 2.1. The table of $S_1(n)$ and its first and second differences is interesting and has a lot of information in it. It would take a long time for students to produce such a table by hand. Consider the following approach, which uses Python. Consider the set of exercises presented and then answer the following questions:
   a. What knowledge or understanding relevant to the lecture's learning objectives might be gained when students use Python to create this table?
   b. What knowledge or understanding relevant to the lecture's learning objectives might be lost?

   Input your responses at the end of this question.

Excercises	The code
1. Ask the students to produce some Python code which prints the number 1 to 10 using k as a variable.	for k in range(1, 11): print(k);

2. Ask the students to produce some Python code which prints the number 1 to n, again using k as a variable. Use this code to print the numbers 1 to 100.	n = 100;
	for k in range(1, n+1):
	print(k);

3. Create a new variable SUM which will accumulate the sum of the numbers 1 + 2 + dots + n. Use this code to print the S1(n) for n = 1 to 100.	n = 100;
	SUM = 0;
	for k in range(1, n+1):
	SUM=SUM + k;
	print(k);

4. To summarize and Exercises 1 to 3, use def sum(m,n) to create a Python function which computes the function Sm(n). Use your code to evaluate sum(1,10) and reproduce the first row of the table in 2.1 Exploration 1.	def sum(m,n):
	SUM = 0;
	for k in range (1,n+1);
	SUM = SUM +
	k**m;
	print(SUM);
	sum(1,10);

5. Explain to yourself or another student, how to produce the row of first differences in the table. Think about what is being added to $S_m(n)$ each time. If you are unsure of your answer, write some Python code to produce the table by creating a function called first_difference(m,n).	def
	first_difference(m,n):
	SUM = 0
	for k in range (1, n+1):
	print(k**m);
	
	first difference (1,10);

a. What knowledge or understanding relevant to the lecture's learning objectives might be gained when students use Python to create this table?

b. What knowledge or understanding relevant to the lecture's learning objectives might be lost?

3. An alternative to creating the table with Python, it's possible to create the table, say in Excel, and give students the Python code to read the file. Students could then write code to work with the table and compute the differences. Execute this strategy.
    a. What coding concepts will students need in order to work with the table and compute differences?

    b. Compare and contrast this approach with the series of exercises described in question 2. What is the same and what is different about how the coding activity could support students' understanding of the mathematics in this lecture series?

c. Create a "walk through" of your code from question 3a). Using the software of your choice, create a screencast recording of your code and output. Include explanations and annotations that would help students make clear connections between the code and the mathematics.
Submit your walk through as a video link.

4. Explore the notion of telescoping with sequences. Generalize how to find finite geometric sums or a rational number by overlaying the repeating pattern. Using Python, find
   a. The rational representation of the fraction $x = 0.147147...$
   b. The sum of $S = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

   Reflect on your experience using Python. What mathematical insights could students gain through this approach? What challenges might they face?

5. Revisit your ideas from question 1. What parts of the lectures might be most beneficial for students to code for themselves? Which problems are best suited for student learning? Please explain.

6. Students may not be aware of what they are doing, or where they are at, when doing math. Develop one of your examples from question 5. What guidance might you need to add in order to make students' mathematics learning explicit?

7. Reflect on how the flow, pacing, and organization of ideas in this lecture series may need to change when shifting teaching approaches from lecture-based delivery to student-centered interactions with technology. Anticipate some of the implications this might have on assessment approaches.